American Swaption Valuation

A model is presented for pricing single-currency, American style fixed-for-floating interest rate swaptions

We consider a single currency swap specified as follows,

- reset point, T_i, for $i = 0,\ldots,M$, where $0 < T_0 < ... < T_M$,
- floating-leg payment, $N_i\ L(T_i;T_i,T_{i+1})\,\Delta_i$, at settlement time, T_{i+1}, for $i = 0,\ldots,M-1$, where
  - N_i is a notional amount,
  - $\Delta_i = T_{i+1} - T_i$,
  - $L(T_i;T_i,T_{i+1}) = \frac{1}{\Delta_i}\left(\frac{1}{P(T_i,T_{i+1})} - 1\right)$ is the simple interest rate[1] applicable at T_i for the accrual period, Δ_i,
- fixed-leg payment, $N_i\ R_i\ \Delta_i$, at settlement time, T_{i+1}, for $i = 0,\ldots,M-1$, where R_i is a simple, annualized rate (ref https://finpricing.com/lib/IrCurveIntroduction.html).

An American style swaption allows the holder to choose the entry point, into the tail of the swap, from a list of possible exercise times (e.g., a window of successive business days). Specifically let t_i and τ_i, for $i = 1,\ldots,n$, where $t_i \le \tau_i$,

$$0 < t_1 < ... < t_n,$$

[1] Here $P(T_i,T_{i+1})$ denotes the price at T_i of a zero-coupon bond with maturity, T_{i+1}, and unit face value.

and

$$T_0 \le \tau_1 < ... < \tau_n \le T_M ,$$

denote a respective notification time and exercise time. Next consider a particular such pair of times, t and τ, and assume that

$$T_i < \tau < T_{i+1}$$

for some $i \in \{0,..., M-1\}$. If notification is given at time t, then the respective floating rate and fixed rate payments,

$$N_j \; L(T_j; T_j, T_{j+1}) \, \Delta_j$$

and

$$N_j \; R_j \; \Delta_j ,$$

must be made at T_{j+1}, for $j = i+1, \ldots, M-1$. In addition a blended Libor rate, $\hat{L}$, is determined at τ, and the respective floating rate and fixed rate payments,

$$N_i \; \hat{L} \left(T_{i+1} - \tau \right)$$

and

$$N_i \; R_i \left(T_{i+1} - \tau \right) ,$$

must be made at time T_{i+1}.

We consider the "*BK*" method for valuing single-currency, fixed-for-floating interest rate, American style swaptions with features of the type described in Section 2. The *BK* method is an implementation of a "disconnected" tree discretization of a one factor Black-Karazinski (BK) risk-neutral short-rate process of the form below.

Let r denote the short-interest rate. We consider a short-interest rate process such that $\log r$ satisfies a risk-neutral SDE of the form,

$$d \log r_t = \left(\theta_t - a_t \log r_t\right) dt + \sigma_t dW_t\,, \tag{3.1}$$

where

- a_t is a piecewise constant mean reversion rate,
- σ_t is a piecewise constant volatility function,
- θ_t is chosen to fit the initial term structure of discount factors,
- W_t is a standard Brownian motion.

A disconnected tree discretization of the short-rate process above is non-recombinant by design, but employs an interpolation scheme to approximate short-rate values at tree nodes along a time slice.

Calibration is accomplished by matching, in a least squares sense, the model price against the market price for each respective European style payer swaption in a cache of calibration securities. The volatility break points are related to the forward start times of the respective swaptions in the calibration portfolio (see Section 4.2 for a typical specification). Given an American swaption,

Consider a particular exercise point, τ, and assume that

$$T_i < \tau < T_{i+1}$$

for some $i \in \{0, ..., M-1\}$. Recall that the effective Libor rate at τ is, in practice, linearly interpolated from certain bracketing Libor rates. We, however, set the effective Libor rate at τ to

$$\hat{L} = \frac{1}{T_{i+1} - \tau}\left(\frac{1}{P(\tau, T_{i+1})} - 1\right).$$

This treatment is computationally efficient, since it avoids determining bracketing Libor rate values.

We note that, although schedules of this form are consistent, We do not allow notification times corresponding to future exercise points to precede previous exercise times; that is, We do not permit respective notification delay and exercise time schedules, $t_1 < ... < t_n$ and $\tau_1 < ... < \tau_n$, such that

$$t_i \leq \tau_{i-1},$$

for some $i \in \{2,..., n\}$. For example, We do not allow a schedule of successive daily exercise times with *non-zero* day notification delay; moreover, there are currently live deals booked without notification delay, but for which the deal confirmation specifies a two-day notification delay. In Section 5 we examine the pricing error introduced by the omission of notification delay for successive daily exercise schedules.